

## Effects of Elastic Energy of Thin Films on Bending of a Cantilevered Magnetostrictive Film-Substrate System

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In this paper, effects of elastic energy of magnetostrictive film on the deflection of a cantilevered film-substrate system are investigated. The total energy including the elastic energy of magnetostrictive film is formulated. And it is minimized to give the curvatures and the position of neutral axis of the cantilevered system. To discuss the effects of the elastic energy of film in a measured system, three magnetostrictive unimorph cantilevers and a bimorph cantilever reported elsewhere are reviewed. It is shown that the assumption, since the thickness of film is much smaller than that of substrate the film elastic energy is negligible, can cause considerable error in evaluating magnetostrictive coefficients. Not the ratio of thicknesses but elastic energies between film and substrate is also shown to play important role in making decision whether the assumption is valid or not.

**Key Words :** Magnetostrictive Film, Magnetostriction, Magnetostrictive Coefficient, Cantilever Actuator

### 1. Introduction

Characterization of a magnetostrictive coefficient of giant magnetostrictive thin films has much importance in future industrial applications. Especially for MEMS actuators utilizing magnetostriction as a principal driving mechanism, more accurate value of magnetostrictive coefficient is needed to evaluate their mechanical performance correctly.

Magnetostrictive coefficients (or magnetoelastic coupling coefficients) can be determined by measuring the deflection of a cantilevered substrate on which a giant magnetostrictive thin film is deposited since the magnetostriction is proportional to the deflection.

In 1976, the relationship between magnetostrictive coefficient and deflection was proposed first (Klokholm, 1976). Energy minimization method was applied to find the relation between curvature and magnetoelastic coupling coefficient (E. du Tremolet and Peuzin, 1994). Elastic equations for the three-dimensional cantilever problem were exactly solved to find the deflection caused by anisotropic magnetostrictive stresses (Riet, 1994). Using energy minimization method and a variable curvature in the cantilever width, relations for the clamped cantilever system were derived (Marcus, 1997). More realistic assumption on the curvature in width direction was made (Iannotti and Lanotte, 1999). The effects of magnetostrictive film thickness on the cantilever bending were theoretically studied (Zhang et al., 2002).

In the previous works based on the energy minimization method (E. du Tremolet and Peuzin, 1994; Marcus, 1997; Iannotti and Lanotte, 1999), the elastic energy of magnetostrictive film are assumed to be much smaller than that of substrate, and thus considered to be negligible. Since

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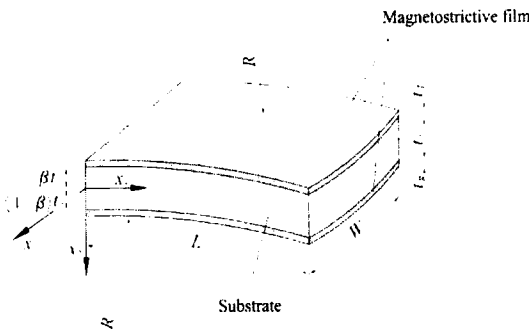
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the film thickness is usually much smaller than the substrate thickness, this assumption has been widely accepted as valid. But when the flexural rigidity of the film is comparable to that of substrate, the assumption is no longer valid. It means that the elastic energy of film must be included in energy minimization method.

In this paper for the cantilever which is sandwiched in between two different magnetostrictive films the total energy including the elastic energy of the films is minimized. Four examples of cantilever are investigated to discuss quantitatively the effects of the elastic energy of the films. Relying upon the results showing that the values of magnetostrictive coefficient calculated by the previous authors are deviated from the actual ones, meaningful conclusions are drawn.

## 2. Total Energy Minimization

As shown in Fig. 1, the cantilevered film-substrate system of length  $L$ , width  $W$ , upper film thickness  $t_f$ , and lower film thickness  $t_g$  is considered. The coordinate axes origin at the unstrained layer at distance  $\beta t_s$  from the interfaces between the substrate and the upper magnetostrictive film.  $R_1$  and  $R_2$  are the curvatures in the  $x_1Ox_3$  and  $x_2Ox_3$  plane respectively. Two magnetostrictive films are considered to have different sign of magnetostriction to reflect practical situation. In this configuration, a unimorph can-



**Fig. 1** A schematic diagram of a cantilevered magnetostrictive film-substrate system

tilever on which single magnetostrictive film only is deposited can be considered by simply assigning the thickness of relevant magnetostrictive film as zero. For magnetization along the length, which is along axis  $x_1$  in Fig. 1, the magnetoelastic energy per unit volume of the films is given by (Marcus, 1997)

$$\tilde{E}^{me} = -\lambda(c_{11} - c_{12}) \left( \epsilon_{11} - \frac{1}{2} \epsilon_{22} - \frac{1}{2} \epsilon_{33} \right) \quad (1)$$

where  $\lambda$  is the magnetostrictive coefficient,  $c_{11}$ ,  $c_{12}$  are cubic elastic constants, and  $\epsilon_{11}$ ,  $\epsilon_{22}$ ,  $\epsilon_{33}$  are the normal strains along the  $x_1$ ,  $x_2$ ,  $x_3$  axes, respectively. The magnetostrictive stresses produced in the three cubic directions are

$$\sigma_{11} = -\frac{\partial \tilde{E}^{me}}{\partial \epsilon_{11}} = \lambda(c_{11} - c_{12}) \quad (2a)$$

$$\sigma_{22} = -\frac{\partial \tilde{E}^{me}}{\partial \epsilon_{22}} = -\frac{\lambda}{2}(c_{11} - c_{12}) \quad (2b)$$

$$\sigma_{33} = -\frac{\partial \tilde{E}^{me}}{\partial \epsilon_{33}} = -\frac{\lambda}{2}(c_{11} - c_{12}) \quad (2c)$$

The stress along axis  $x_3$  in the magnetostrictive film is given by  $\sigma_{33}$  in Eq. (2c) and the stress along axis  $x_3$  vanishes in the substrate. With the elastic equations and Eq. (2c), the strain  $\epsilon_3$  is expressed

$$\epsilon_{33} = -\frac{\lambda}{2c_{11}}(c_{11} - c_{12}) - \frac{c_{12}}{c_{11}}(\epsilon_{11} + \epsilon_{22}) \quad (3)$$

where  $\lambda$  is zero in the substrate. Eliminating the strain  $\epsilon_3$  in Eq. (1) by using Eq. (3) gives

$$\tilde{E}_f^{me} = -\frac{\lambda_f E_f}{2(1-\nu_f^2)} \{ (2-\nu_f) \epsilon_{11} - (1-2\nu_f) \epsilon_{22} \} - \frac{\lambda_f^2 E_f (1-2\nu_f)}{4(1-\nu_f^2)} \quad (4)$$

$$\tilde{E}_g^{me} = -\frac{\lambda_g E_g}{2(1-\nu_g^2)} \{ (2-\nu_g) \epsilon_{11} - (1-2\nu_g) \epsilon_{22} \} - \frac{\lambda_g^2 E_g (1-2\nu_g)}{4(1-\nu_g^2)} \quad (5)$$

where  $E$  is the Young's modulus,  $\nu$  the Poisson's ratio,  $\lambda$  the magnetostrictive coefficient, and  $\epsilon_{11}$ ,  $\epsilon_{22}$ ,  $\epsilon_{33}$  are the normal strains along the  $x_1$ ,  $x_2$ ,  $x_3$  axes, respectively. The subscripts  $f$  and  $g$  indicate the top magnetostrictive film and bottom one, respectively.

The elastic energies per unit volume stored in the films and the substrate are given by (Marcus, 1997)

$$\begin{aligned} \tilde{E}_f^{el} = & \frac{1}{2} c_{11} (\epsilon_{11}^2 + \epsilon_{22}^2 + \epsilon_{33}^2) \\ & + c_{12} (\epsilon_{22}\epsilon_{33} + \epsilon_{33}\epsilon_{11} + \epsilon_{11}\epsilon_{22}) \end{aligned} \quad (6)$$

Eliminating the strain  $\epsilon_3$  in Eq. (6) by using Eq. (3) gives

$$\begin{aligned} \tilde{E}_f^{el} = & \frac{E_f}{2(1-\nu_f^2)} (\epsilon_{11}^2 + \epsilon_{22}^2 + 2\nu_f\epsilon_{11}\epsilon_{22}) \\ & + \frac{E_f\lambda_f^2(1-2\nu_f)}{8(1-\nu_f^2)} \end{aligned} \quad (7)$$

$$\begin{aligned} \tilde{E}_g^{el} = & \frac{E_g}{2(1-\nu_g^2)} (\epsilon_{11}^2 + \epsilon_{22}^2 + 2\nu_g\epsilon_{11}\epsilon_{22}) \\ & + \frac{E_g\lambda_g^2(1-2\nu_g)}{8(1-\nu_g^2)} \end{aligned} \quad (8)$$

$$\tilde{E}_s^{el} = \frac{E_s}{2(1-\nu_s^2)} (\epsilon_{11}^2 + \epsilon_{22}^2 + 2\nu_s\epsilon_{11}\epsilon_{22}) \quad (9)$$

where the subscript  $s$  indicates the substrate. For small bending deflection, the strain can be assumed to be linear in  $x_3$ .

$$\epsilon_{11} = -\frac{x_3}{R_1} \equiv -\frac{x_3\alpha_1}{t_s} \quad (10)$$

$$\epsilon_{22} = -\frac{x_3}{R_2} \equiv -\frac{x_3\alpha_2}{t_s} \quad (11)$$

where  $\alpha_1 \equiv -t_s/R_1$  and  $\alpha_2 \equiv -t_s/R_2$  are dimensionless parameters of curvatures along  $x_1$  and  $x_2$ , respectively.

Substituting Eq. (10) and Eq. (11) into Eqs. (4) ~ (5), and (7) ~ (9) and integrating over the relevant volume give

$$\begin{aligned} E_f^{pe} = & -\frac{V_f\lambda_f E_f}{2(1-\nu_f^2)} \left( \beta + \frac{t_f}{t_s} \right) \{ (2-\nu_f)\alpha_1 - (1-2\nu_f)\alpha_2 \} \\ & - \frac{V_f\lambda_f^2 E_f (1-2\nu_f)}{4(1-\nu_f^2)} \end{aligned} \quad (12)$$

$$\begin{aligned} E_g^{pe} = & \frac{V_g\lambda_g E_g}{2(1-\nu_g^2)} \left\{ -(1-\beta) - \frac{t_g}{2t_s} \right\} \{ (2-\nu_g)\alpha_1 - (1-2\nu_g)\alpha_2 \} \\ & - \frac{V_g\lambda_g^2 E_g (1-2\nu_g)}{4(1-\nu_g^2)} \end{aligned} \quad (13)$$

$$E_s^{el} = \frac{V_s E_s}{2(1-\nu_s^2)} (\alpha_1^2 + \alpha_2^2 + 2\nu_s\alpha_1\alpha_2) \left( \beta^2 - \beta + \frac{1}{3} \right) \quad (14)$$

$$\begin{aligned} E_f^{el} = & \frac{V_f E_f}{2(1-\nu_f^2)} (\alpha_1^2 + \alpha_2^2 + 2\nu_f\alpha_1\alpha_2) \left\{ \beta^2 + \beta \left( \frac{t_f}{t_s} \right) + \frac{1}{3} \left( \frac{t_f}{t_s} \right)^2 \right\} \\ & + \frac{V_f E_f \lambda_f^2 (1-2\nu_f)}{8(1-\nu_f^2)} \end{aligned} \quad (15)$$

$$\begin{aligned} E_g^{el} = & \frac{V_g E_g}{2(1-\nu_g^2)} (\alpha_1^2 + \alpha_2^2 + 2\nu_g\alpha_1\alpha_2) \left\{ (1-\beta)^2 + (1-\beta) \left( \frac{t_g}{t_s} \right) + \frac{1}{3} \left( \frac{t_g}{t_s} \right)^2 \right\} \\ & + \frac{V_g \lambda_g E_g (1-2\nu_g)}{8(1-\nu_g^2)} \end{aligned} \quad (16)$$

where  $V$  is the volume,  $t$  is the thickness, and  $\beta$  is a dimensionless value of the position of the neutral axis. The total energy which is the sum of Eqs. (12) ~ (16) should be minimized to ensure equilibrium condition. This is done by setting the partial derivative of total energy with respect to three independent variables,  $\alpha_1$ ,  $\alpha_2$  and  $\beta$ , to zero.

$$\left( 2P + \frac{2DM}{\lambda_f} + \frac{2EN}{\lambda_g} \right) \alpha_1 + \left( 2\nu_s P + \frac{2DM\nu_f}{\lambda_f} + \frac{2EN\nu_g}{\lambda_g} \right) \alpha_2 \quad (17)$$

$$= D \left( \beta + \frac{t_f}{2t_s} \right) (2-\nu_f) + E \left\{ -(1-\beta) - \frac{t_g}{2t_s} \right\} (2-\nu_g)$$

$$\left( 2\nu_s P + \frac{2DM\nu_f}{\lambda_f} + \frac{2EN\nu_g}{\lambda_g} \right) \alpha_1 + \left( 2P + \frac{2DM}{\lambda_f} + \frac{2EN}{\lambda_g} \right) \alpha_2 \quad (18)$$

$$= -D \left( \beta + \frac{t_f}{2t_s} \right) (2-\nu_f) - E \left\{ -(1-\beta) - \frac{t_g}{2t_s} \right\} (1-2\nu_g)$$

$$-D \{ (2-\nu_f)\alpha_1 - (1-2\nu_f)\alpha_2 \} - E \{ (2-\nu_g)\alpha_1 - (1-2\nu_g)\alpha_2 \} \\ + (\alpha_1^2 + \alpha_2^2 + 2\nu_s\alpha_1\alpha_2) (2\beta - 1)$$

$$+ \frac{D}{\lambda_f} (\alpha_1^2 + \alpha_2^2 + 2\nu_f\alpha_1\alpha_2) \left\{ 2\beta + \left( \frac{t_f}{t_s} \right) \right\} \quad (19)$$

$$+ \frac{E}{\lambda_g} (\alpha_1^2 + \alpha_2^2 + 2\nu_g\alpha_1\alpha_2) \left\{ -2(1-\beta) - \left( \frac{t_g}{t_s} \right) \right\} = 0$$

In Eqs. (17) ~ (19), the following constants are used for the sake of the simplicity.

$$A = \frac{V_f E_f}{(1-\nu_f^2)}, \quad B = \frac{V_g E_g}{(1-\nu_g^2)}, \quad C = \frac{V_s E_s}{(1-\nu_s^2)},$$

$$E = \frac{B}{C} \lambda_g = \frac{V_g E_g (1-\nu_s^2)}{V_s E_s (1-\nu_g^2)} \lambda_g, \quad P = \beta^2 - \beta + \frac{1}{3},$$

$$M = \beta^2 + \beta \left( \frac{t_f}{t_s} \right) + \frac{1}{3} \left( \frac{t_f}{t_s} \right)^2,$$

$$N = (1-\beta)^2 + (1-\beta) \left( \frac{t_g}{t_s} \right) + \frac{1}{3} \left( \frac{t_g}{t_s} \right)^2$$

We can obtain the curvatures,  $\alpha_1$ ,  $\alpha_2$ , and the  $x_2$  coordinates of neutral axis,  $\beta$ , for magnetization along the length by solving Eqs. (17) ~ (19) simultaneously. The curvatures,  $\alpha_1$ ,  $\alpha_2$ , for magnetization along the length are designate  $\alpha_1^t$ ,  $\alpha_2^t$  to distinguish them from those  $\alpha_1^w$ ,  $\alpha_2^w$  for

magnetization along the width. The curvatures for magnetization along the width are given by interchange of  $\alpha_1$  and  $\alpha_2$ .

$$\alpha_1^w = \alpha_2^i \quad \alpha_2^w = \alpha_1^i \quad (20)$$

Thus, the deflection difference measured under rotating magnetic field is

$$\delta = \frac{L^2}{2t_s} (\alpha_1^i - \alpha_1^w) \quad (21)$$

### 2.1 Unimorph cantilever

Under the assumptions that the elastic energy stored in magnetostrictive film is much smaller than one in substrate, Eqs. (17) ~ (19) are reduced to Eqs. (22) ~ (24) for unimorph cantilever.

$$(2P)\alpha_1 + (2P\nu_s)\alpha_2 = D\beta(2 - \nu_f) \quad (22)$$

$$(2\nu_s P)\alpha_1 + (2P)\alpha_2 = -D\beta(1 - 2\nu_f) \quad (23)$$

$$\begin{aligned} & -D\{(2 - \nu_f)\alpha_1 - (1 - 2\nu_f)\alpha_2\} \\ & + (\alpha_1^i + \alpha_2^i + 2\nu_s\alpha_1\alpha_2)(2\beta - 1) = 0 \end{aligned} \quad (24)$$

Solving Eqs. (22) ~ (24) for the  $\alpha_1$ ,  $\alpha_2$ , and  $\beta$  gives (Marcus, 1997)

$$\alpha_1^i = \frac{6t_f E_f}{t_s E_s} \frac{\{(1 - \nu_f\nu_s) + 0.5(\nu_s - \nu_f)\}}{1 - \nu_f^2} \lambda_f \quad (25)$$

$$\alpha_2^w = -\frac{6t_f E_f}{t_s E_s} \frac{\{0.5(1 - \nu_f\nu_s) + \nu_s - \nu_f\}}{1 - \nu_f^2} \lambda_f \quad (26)$$

$$\beta = \frac{2}{3} \quad (27)$$

When the elastic energy stored in magnetostrictive film is comparable to one in substrate, Eqs. (17) ~ (19) are reduced to Eqs. (28) ~ (30)

$$\begin{aligned} & \left(2P + \frac{2DM}{\lambda_f}\right)\alpha_1 + \left(2P\nu_s + \frac{2DM\nu_f}{\lambda_f}\right)\alpha_2 \\ & = D\left(\beta + \frac{t_f}{2t_s}\right)(2 - \nu_f) \end{aligned} \quad (28)$$

$$\begin{aligned} & \left(2P\nu_s + \frac{2DM\nu_f}{\lambda_f}\right)\alpha_1 + \left(2P + \frac{2DM}{\lambda_f}\right)\alpha_2 \\ & = -D\left(\beta + \frac{t_f}{2t_s}\right)(1 - 2\nu_f) \end{aligned} \quad (29)$$

$$\begin{aligned} & -D\{(2 - \nu_f)\alpha_1 - (1 - 2\nu_f)\alpha_2\} \\ & + (\alpha_1^i + \alpha_2^i + 2\nu_s\alpha_1\alpha_2)(2\beta - 1) \\ & + \frac{D}{\lambda_f}(\alpha_1^i + \alpha_2^i + 2\nu_f\alpha_1\alpha_2)\left\{2\beta + \left(\frac{t_f}{t_s}\right)\right\} = 0 \end{aligned} \quad (30)$$

Since Eqs. (28) ~ (30) are very complicate, it is hardly possible to obtain the closed form solutions of  $\alpha_1$ ,  $\alpha_2$ , and  $\beta$ . Numerical method should be used to calculate them. It is worth to notice that the position of neutral axis is no longer equal to 2/3. It depends on the thickness and magnetostrictive coefficient.

### 2.2 Bimorph cantilever

When the elastic energy stored in magnetostrictive film is neglected, Eqs. (17) ~ (19) are reduced to Eqs. (31) ~ (33).

$$\begin{aligned} & (2P)\alpha_1 + (2P\nu_s)\alpha_2 \\ & = D\beta(2 - \nu_f) + E(\beta - 1)(2 - \nu_g) \end{aligned} \quad (31)$$

$$\begin{aligned} & (2P\nu_s)\alpha_1 + (2P)\alpha_2 \\ & = -D\beta(1 - 2\nu_f) - E(\beta - 1)(1 - 2\nu_g) \end{aligned} \quad (32)$$

$$\begin{aligned} & -D\{(2 - \nu_f)\alpha_1 - (1 - 2\nu_f)\alpha_2\} \\ & - E\{(2 - \nu_g)\alpha_1 - (1 - 2\nu_g)\alpha_2\} \\ & + (\alpha_1^i + \alpha_2^i + 2\nu_s\alpha_1\alpha_2)(2\beta - 1) = 0 \end{aligned} \quad (33)$$

Solving for the  $\alpha_1$ ,  $\alpha_2$ , and  $\beta$  gives (Marcus, 1997)

$$\alpha_1 = 6 \frac{V_f E_f}{V_s E_s} \frac{1 - \nu_s\nu_f + \frac{1}{2}(\nu_s - \nu_f)}{1 - \nu_f^2} \lambda_f \quad (34)$$

$$-6 \frac{V_g E_g}{V_s E_s} \frac{1 - \nu_s\nu_g + \frac{1}{2}(\nu_s - \nu_g)}{1 - \nu_g^2} \lambda_g$$

$$\alpha_2 = -6 \frac{V_f E_f}{V_s E_s} \frac{\frac{1}{2}(1 - \nu_s\nu_f) + \nu_s - \nu_f}{1 - \nu_f^2} \lambda_f \quad (35)$$

$$+ 6 \frac{V_g E_g}{V_s E_s} \frac{\frac{1}{2}(1 - \nu_s\nu_g) + \nu_s - \nu_g}{1 - \nu_g^2} \lambda_g$$

$$\beta = 1/2 \quad (36)$$

For the case when the elastic energies of the film is considered, Eqs. (17) ~ (19) should be solved simultaneously. The position of neutral axis is not 1/2 any longer and changes with the thickness and magnetostrictive coefficient.

## 3. Discussion

In order to discuss the effects of elastic energy

of the magnetostrictive film in quantitative manner, the examples that information on the magnetostrictive coefficient is available are reviewed.

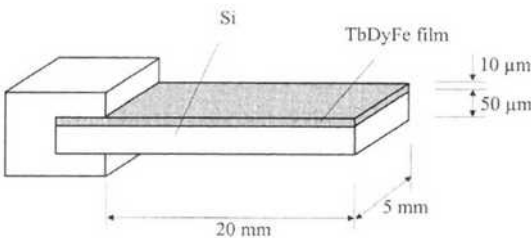
**3.1 Unimorph cantilever**

The magnetostrictive cantilever fabricated by Quandt et al., shown in Fig. 2, is firstly considered. The Young's modulus and Poisson's ratio are taken from reference (Body, 1997) and listed in Table 1. It should be noticed that the ratio of Young's moduli between the film and substrate is about 3. When the magnetic field was applied along the length direction, the deflections of the cantilever were measured, as seen in Fig. 3, by Quandt et al. Using the measured displacements, the magnetostrictive coefficient of TbDyFe film is calculated in two different cases : case for neglecting the elastic energy of the film and for considering it. Figure 4 shows the calculated magnetostrictive coefficients. As the magnetic field increases, differences between two cases became larger. As expected, when the elastic energy is considered, magnetostrictive coefficients are evaluated higher about 10%.

As a second example, the magnetostrictive actuator coated with the Tb-Fe film having positive magnetostriction or the Sm-Fe film having negative one (Honda et al., 1994), shown in

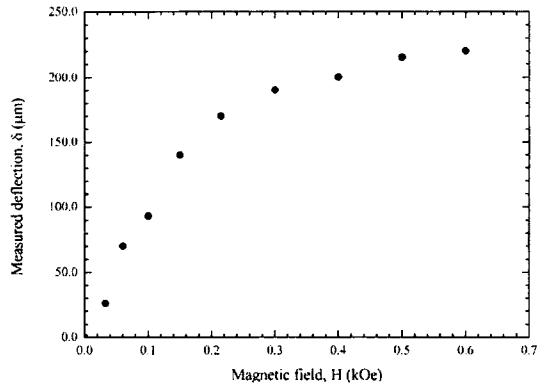
**Table 1** Mechanical properties used in simulation of the cantilever fabricated by Quandt et al. (1994)

Properties	TbDyFe film	Si substrate
Modulus of elasticity, GPa	50.0	169.0
Poisson's ratio	0	0.067

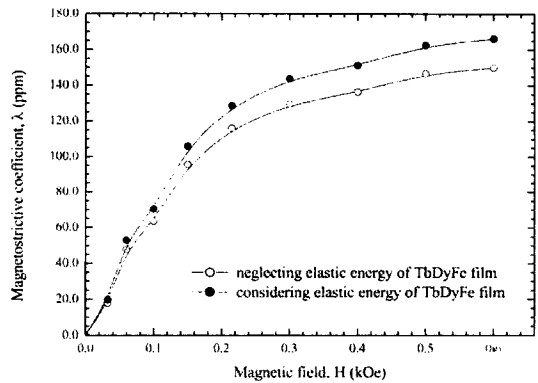


**Fig. 2** A schematic view of the magnetostrictive thin film cantilever (Quandt et al., 1994)

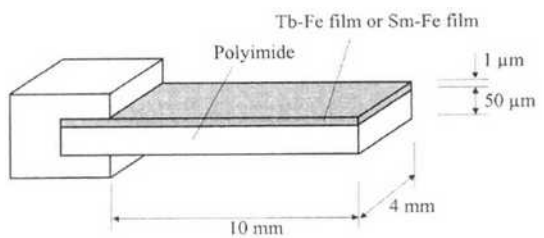
Fig. 5, was considered. Different from the cantilever by Quandt et al., polyimide which is more flexible than silicon is used as substrate material in this application. Unfortunately the data on mechanical properties was not reported. Thus, values for materials having similar chemical composition, listed in Table 2, are used instead (Shima et al., 1997, ANSYS). Similar to



**Fig. 3** Measured deflections of a unimorph cantilever on which the TbDyFe film was coated



**Fig. 4** Calculated magnetostrictive coefficients of TbDyFe film



**Fig. 5** A schematic view of the magnetostrictive thin film cantilever (Honda et al., 1994)

the first example, the magnetostrictive coefficients are calculated with the experimental data shown Fig. 6 for Tb-Fe film and Sm-Fe film respectively. Figure 7 shows the calculated magnetostrictive coefficients for Tb-Fe film. It can be seen that there are considerable differences, nearly two times, between two calculation results. It means that the effect of elastic energy of film can't be neglected. As seen in Fig. 8, such differences are also observed for Sm-Fe film. The results for three examples reviewed are summarized in Table 3. To discuss the results in detail, following measures are defined.

**Table 2** Mechanical properties used in simulation of the cantilever fabricated by Honda et al. (1994)

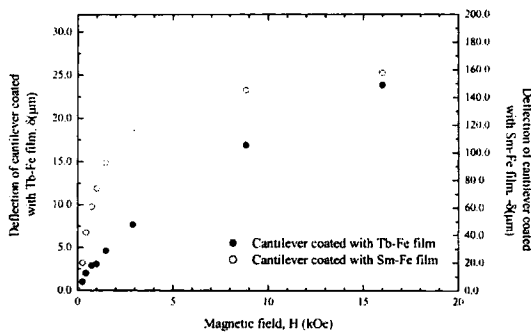
Properties	Tb-Fe film*	Sm-Fe film*	Polyimide**
Modulus of elasticity, GPa	76	40	7.5
Poisson's ratio	0.3	0.3	0.35

\*taken from (Shima et al., 1997)

\*\*taken from ANSYS.

**Table 3** Summary of calculation results

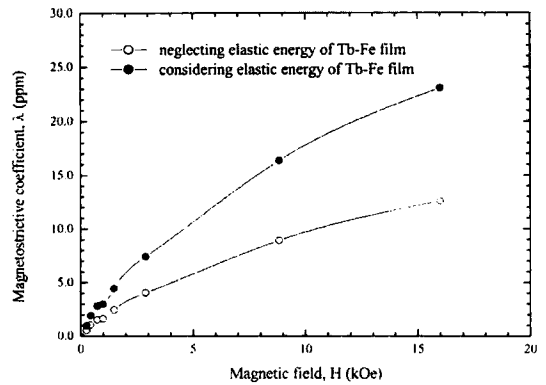
	TbDyFe/Si	Tb-Fe/ Polyimide	Sm-Fe/ Polyimide
$\xi$ , %	20	2	2
$\varsigma$ , %	0.52	19.5	10.3
$\eta$ , %	9.8	45.4	29.9
$\beta$	0.6392	0.6634	0.6634



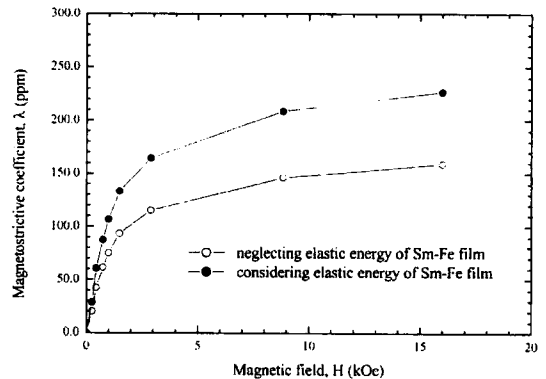
**Fig. 6** Measured deflections of a unimorph cantilever on which the Tb-Fe film or the Sm-Fe film was coated

$$\xi = \frac{t_f}{t_s} \times 100, \varsigma = \frac{t_f}{t_s} \frac{E_f}{E_s} \frac{1 - \nu_s^2}{1 - \nu_f^2} \times 100, \eta = \frac{\lambda_c - \lambda_n}{\lambda_c} \times 100 \quad (37)$$

where  $\xi$  are the ratio of the film thickness to the substrate thickness,  $\varsigma$  the ratio of the film elastic energy to the substrate elastic energy, and  $\eta$  is the error of magnetostrictive coefficient.  $\lambda_c$  and  $\lambda_n$  are magnetostrictive coefficients when the elastic energy of film is considered and neglected respectively. For TbDyFe/Si, although the parameter  $\xi$  is relatively large, the small error of 9.8% is caused by neglecting the effects of film elastic energy. For the other cases, on the contrary, although the film thickness is much smaller than that of substrate, significant errors are induced. We can also notice that for all cases there



**Fig. 7** Calculated magnetostrictive coefficients of Tb-Fe film

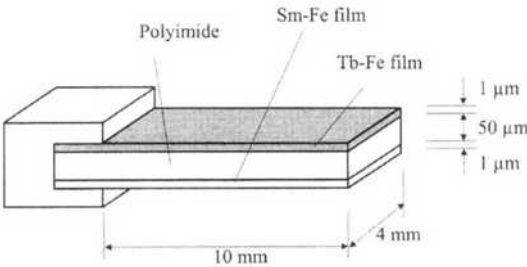


**Fig. 8** Calculated magnetostrictive coefficients of Sm-Fe film

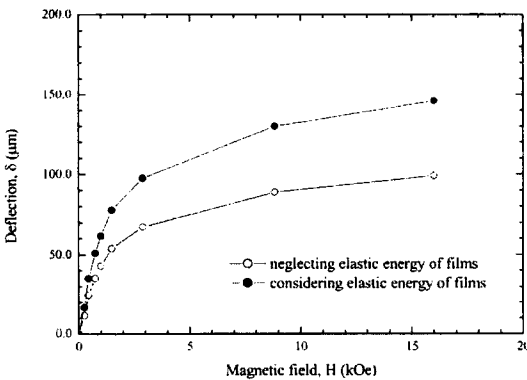
are direct relationship between the parameter,  $\zeta$ , and error,  $\eta$ . Consequently, we can conclude that the ratio of elastic energies between film and substrate is a key parameter in making decision whether the assumption that the film elastic energy is negligible is valid or not. It is appear that the value  $\beta$  presenting the position of neutral axis is slightly different from  $2/3$ . [see Eq. (27)]

**3.2 Bimorph cantilever**

From the previous section, we ascertain that if the film elastic energy is neglected the magnetostrictive coefficient become under-estimated. In this section, the effects of under-estimated magnetostrictive coefficients in performance evaluation of magnetostrictive MEMS actuators are investigated. The bimorph cantilever actuator shown in Fig. 9 consisting is consisting of the Tb-Fe film having positive magnetostriction and the Sm-Fe film having negative one. With the results in Fig. 7 and 8, the deflections of cantilever are predicted. Figure 10 shows the deflec-



**Fig. 9** Bimorph cantilever actuator (Honda et al., 1994)



**Fig. 10** Deflections of a bimorph cantilever

tions of bimorph cantilever. In this example, the larger deflections are expected when the magnetostrictive coefficient obtained by considering the film elastic energy are used. Maximum differences in displacements reach to  $47 \mu\text{m}$ . The value of  $\beta$  is 0.3487. It is different from  $1/2$ . [see Eq. (36)]

**4. Conclusions**

The effects of film elastic energy on the evaluation of magnetostrictive coefficient are investigated. Experimental examples are considered to discuss it quantitatively. In order to make valid the assumption that the film elastic energy is negligible, the ratio of elastic energies between film and substrate should be very small. The condition that the thickness of the film is smaller than that of substrate can't guarantee the validity of the assumption. It is ascertained that if the film elastic energy is neglected the magnetostrictive coefficient is considerably under-estimated. Finally if the under-estimated magnetostrictive coefficients are used in modeling magnetostrictive actuator, significantly different elastic behaviors from actual one are predicted.

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